Prediction of the Forming Limit Band for Steel Sheets using a new Formulation of Hora's Criterion (MMFC)

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Abstract. The paper analyzes the dispersion of the mechanical parameters and its influence on the forming limit curves of sheet metals. The tests have been made for the case of the DC01 steel sheets. The dispersion of the mechanical parameters has been observed during the experimental research. On the basis of this dispersion, a forming limit band has been calculated using an alternate formulation of Hora's model (MMFC).

Keywords: Plasticity, Sheet metals, Formability, Mechanical parameters.
PACS: 62.20.fq, 81.05.Bx, 81.40.Np, 87.15.La

1. INTRODUCTION

The Forming Limit Diagram (FLD) represents an efficient tool to characterize the formability of sheet metals [1]. The FLD is a curve relating pairs of principal limit strains, which can be obtained at the surface of the sheet metal during a forming process prior to the occurrence of some defects (necking, fracture, etc.). The first theoretical FLD models were based on the diffuse and localized necking theories proposed by Hill [2] and Swift [3], respectively. Since then, several other mathematical models have been developed. One of those theories has been proposed by Hora [4]. This paper presents an alternate formulation of Hora's model.

The FLD predictions are strongly influenced by the shape of the yield locus used in the computational model. In 1948, Hill proposed the first yield criterion for anisotropic materials [1]. The mathematical shape of the criterion is a simple quadratic function and the coefficients can be analytically computed. Due to this characteristic, it is the most used plasticity model. Since then, many other yield criteria have been proposed aiming to improve the fitting with experimental data [1].

The experimental research shows that the mechanical parameters are not rigorously constant. Knowing the variability of these material characteristics would allow performing numerical simulations not only for the average values, but also for the extreme ones. After the experimental determination of the mechanical parameters, a study related to the influence of this variability upon FLC can be performed. Having this information acquired, a forming limit band can be calculated.

The aim of this paper is to determine the forming limit band for the DC01 steel alloy, based on the dispersion of the uniaxial yield stresses and anisotropic coefficients determined at 0°, 45° and 90° with respect to the rolling direction. The experimental limit strains have been determined by bulging and punch stretching tests.

The computed FLD has been obtained using an alternate formulation of Hora's model. The anisotropic behavior of the sheet metal has been described by the Hill 1948 yield criterion and Swift's hardening law [1]. The comparison between the theoretical and the experimental data shows a good agreement.

2. FLD MODEL

Throughout this paper, the sheet metal is considered to behave as an orthotropic membrane under the plane-stress conditions

$$\begin{aligned} &\sigma_{i3} = \sigma_{3i} = 0, \quad i = 1,2,3, \\ &\dot{\varepsilon}_{j3} = \dot{\varepsilon}_{3j} = 0, \quad j = 1,2, \end{aligned} \tag{1}$$

involving the stresses and strain-rates expressed in the orthotropy frame (1, 2 and 3 are the indices associated to the rolling, transverse, and normal directions, respectively). We also assume that the external loads do not produce tangential stresses and strains:

$$\sigma_{12} = \sigma_{21} = 0, \quad \dot{\varepsilon}_{12} = \dot{\varepsilon}_{21} = 0. \tag{2}$$

The non-zero stresses and strain-rates thus become eigenvalues. In order to emphasize their significance, the following notations will be used: $\dot{\varepsilon}_i = \dot{\varepsilon}_{ii}\left(i = 1,2,3\right)$ – principal strain rates, and $\sigma_j = \sigma_{jj}\left(j = 1,2\right)$ – principal stresses.

The mechanical response of the sheet metal will be described by a rigid-plastic model. The main ingredient of the constitutive model is the yield criterion:

$$\bar{\sigma}\left(\sigma_1, \sigma_2\right) = Y\left(\bar{\varepsilon}\right). \tag{3}$$

where $\bar{\sigma} = \bar{\sigma}\left(\sigma_1, \sigma_2\right) \geq 0$ – equivalent stress (homogeneous function of the first degree), $\bar{\varepsilon} \geq 0$ – equivalent strain, and ${}^tY = {}^tY\left({}^t\bar{\varepsilon}\right) > 0$ – yield parameter controlled by a strictly increasing hardening law. The principal strain-rates are defined by the flow rule

$$\dot{\varepsilon}_j = \dot{\bar{\varepsilon}} \frac{\partial \bar{\sigma}}{\partial \sigma_j}, \quad j = 1,2, \tag{4}$$

and the incompressibility constraint

$$\dot{\varepsilon}_1 + \dot{\varepsilon}_2 + \dot{\varepsilon}_3 = 0. \tag{5}$$

In order to preserve the simplicity of the model, we assume that the local state of the sheet metal evolves along linear load paths subjected to the constraint

$$\alpha = \sigma_2 / \sigma_1 = \text{const.}, \qquad \sigma_1 > 0, \quad \sigma_1 \geq \sigma_2. \tag{6}$$

For any load state having the property given by Eq. (6), $\bar{\sigma}$ and its partial derivatives with respect to the non-zero principal stresses could be expressed as follows:

$$\bar{\sigma} = \sigma_1 f(\alpha), \quad \frac{\partial \bar{\sigma}}{\partial \sigma_1} = g(\alpha), \quad \frac{\partial \bar{\sigma}}{\partial \sigma_2} = h(\alpha). \tag{7}$$

Eq. 7 results from the first-degree homogeneity of the equivalent stress. The functions f, g, and h are only related to the particular formulation of the equivalent stress adopted in the model. For example, if $\bar{\sigma}$ defined by Hill 1948 formula is used [1],

$$\bar{\sigma}^2 = \sigma_1^2 - \frac{2 r_0}{r_0 + 1} \sigma_1 \sigma_2 + \frac{r_0 (r_{90} + 1)}{r_{90} (r_0 + 1)} \sigma_2^2 \tag{8}$$

(r_0 and r_{90} being the anisotropy coefficients associated to the rolling and transverse directions, respectively), f, g, and h can be expressed in the following manner:

$$f(\alpha) = \sqrt{1 - \frac{2 r_0}{r_0 + 1} \alpha + \frac{r_0 (r_{90} + 1)}{r_{90} (r_0 + 1)} \alpha^2}, \tag{9}$$

$$g(\alpha) = \frac{1}{f(\alpha)} \left(1 - \frac{r_0}{r_0 + 1} \alpha \right), \quad h(\alpha) = \frac{1}{f(\alpha)} \frac{r_0}{r_0 + 1} \left(\frac{r_{90} + 1}{r_{90}} \alpha - 1 \right). \tag{10}$$

Eq. 7 allows rewriting the yield criterion and flow rule in the form (see Eqs. 3 and 4):

$$\sigma_1 = Y(\bar{\varepsilon}) / f(\alpha), \tag{11}$$

$$\dot{\varepsilon}_1 = \dot{\bar{\varepsilon}} g(\alpha), \quad \dot{\varepsilon}_2 = \dot{\bar{\varepsilon}} h(\alpha). \tag{12}$$

One may prove that, under the constraint given by Eq. 6, the strain path is also linear. As a consequence, Eq. 12 can be easily integrated with respect to the time variable:

$$\varepsilon_1 = \bar{\varepsilon} g(\alpha), \quad \varepsilon_2 = \bar{\varepsilon} h(\alpha). \tag{13}$$

The FLD model used in this paper has been developed by modifying Hora's strain localization criterion [4]. The new model is expressed as follows:

$$\frac{\partial \sigma_1}{\partial \bar{\varepsilon}} \frac{\partial \bar{\varepsilon}}{\partial \varepsilon_1} + \frac{\partial \sigma_1}{\partial \gamma(\varepsilon_1, \alpha)} \frac{\partial \gamma(\varepsilon_1, \alpha)}{\partial \varepsilon_1} = \sigma_1, \tag{14}$$

where $\gamma(\varepsilon_1, \alpha)$ is a measure of the "distance" separating the current state of the material from the plane-strain. The scalar quantity $\gamma(\varepsilon_1, \alpha)$ is defined by integrating the elementary arc-length of the normalized yield locus:

$$\mathrm{d}\, s = \sqrt{\left(\mathrm{d}\frac{\sigma_1}{Y}\right)^2 + \left(\mathrm{d}\frac{\sigma_2}{Y}\right)^2}. \tag{15}$$

On the basis of the experimental evidence showing that the strain localization is preceded by the evolution of the material towards the plane-strain, the "distance" parameter $\gamma(\varepsilon_1, \alpha)$ is defined in the following manner:

$$\gamma(\varepsilon_1, \alpha) = \frac{1}{\varepsilon_1} \int_{\alpha_{FLC_0}}^{\alpha} \frac{\sqrt{g^2 + h^2}}{f^2} \mathrm{d}\underline{\alpha}. \tag{16}$$

After some mathematical manipulations, Eq. 14 can be rewritten in the form

$$Y = \frac{1}{g}\left(\frac{\mathrm{d}Y}{\mathrm{d}\bar{\varepsilon}} + \frac{Y}{\bar{\varepsilon}} \frac{f\, h}{\sqrt{g^2 + h^2}} \int_{\alpha_{FLC_0}}^{\alpha} \frac{\sqrt{g^2 + h^2}}{f^2} \mathrm{d}\underline{\alpha}\right). \tag{17}$$

This relationship allows the calculation of the equivalent strain associated to necking. As soon as $\bar{\varepsilon}$ is known, the corresponding principal strains result from Eq. 13.

3. EXPERIMENTAL DETERMINATION OF THE MECHANICAL PARAMETERS

The dispersion of the mechanical parameters has been analyzed for the case of a DC01 steel sheet, with a nominal thickness of 0.7 mm. In order to establish the mechanical parameters of the DC01 sheets, uniaxial tensile tests have been performed along three directions corresponding to 0°, 45° and 90° angles measured from the rolling direction. As the main objective of the investigation consisted in analyzing the dispersion of the material characteristics, the specimens have been cut from different sheet areas. Table 1 shows the average values of the mechanical parameters, as well as their minimum and maximum values. Table 2 lists the material constants included in Swift's hardening law:

$$Y = K\left(\varepsilon_0 + \bar{\varepsilon}\right)^n. \tag{18}$$

TABLE 1. Experimental data obtained from tensile tests (DC01 steel sheet, 0.7 mm thickness)

Reference angle	Mechanical parameter	Minimum value	Average value	Maximum value
0°	Y_0 [MPa]	195	211.6	238
	r_0	1.14	1.41	1.65
45°	Y_{45} [MPa]	216	223	230
	r_{45}	1.05	1.17	1.30
90°	Y_{90} [MPa]	213	216.7	221
	r_{90}	1.35	1.92	2.40

TABLE 2. Coefficients of Swit's hardening law (DC01 steel sheet, 0.7 mm thickness)

Material constant	Minimum value	Average value	Maximum value
K [MPa]	–	615.86	–
ε_0	–	0.006	–
n	0.18	0.21	0.22

4. CALCULATION OF THE FORMING LIMIT BAND

Due to the dispersion of the mechanical parameters, the limit strains of the sheet metals spread between an upper and a lower boundary defining a forming limit band. In order to calculate the band, the influence of each mechanical parameter should be studied. In the case of the DC01 steel sheet, the computational tests have shown that increased values of the parameters n, r_0, Y_0, and Y_{90} cause the raising of the limit strains. On the other hand, an increased value of the parameter r_{90} will reduce the formability. The forming limit band shown in Figure 1 has been calculated taking into account these aspects. The computation has been performed using the alternate formulation of Hora's model coupled with the Hill 1948 formulation of the equivalent stress and Swift's hardening law (see the previous sections of the paper). In order to evaluate the performances of the necking criterion, some experimental points have been also plotted on the diagram shown in Figure 1. These points represent limit strains determined through bulging and punch stretching tests.

As one may notice from Figure 1, the predictions of the necking model are in good agreement with the experimental data. The dispersion of the mechanical parameters is more influent at the level of the right branch of the forming limit band. In this region, the model underestimates the spreading of the limit strains. The inaccuracy is probably a consequence of the fact that the identification procedure of Hill 1948 equivalent stress only involves the mechanical parameters associated to uniaxial stress states. The predictive capabilities of the necking model can be improved if using non-quadratic formulations of the equivalent stress with more complex identification procedures (based on uniaxial and biaxial experimental data).

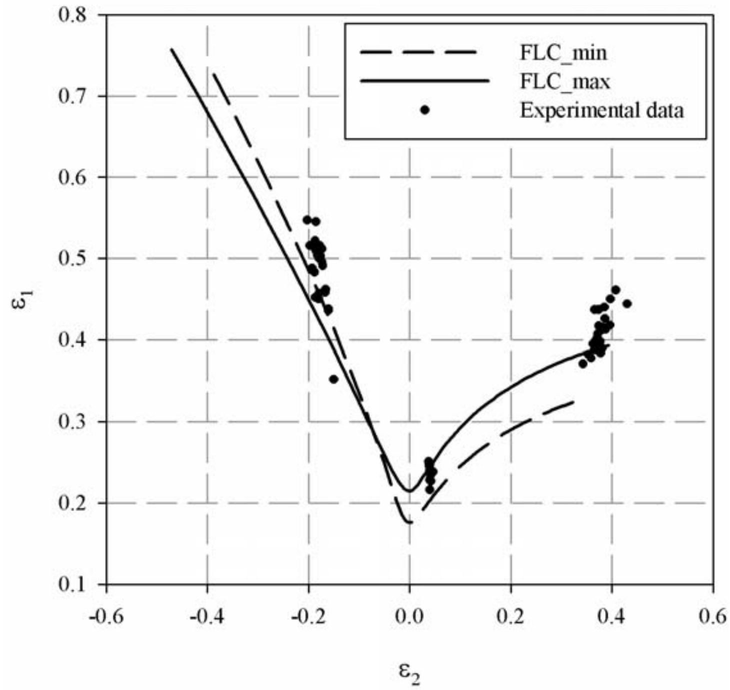

FIGURE 1. Forming limit band (DC01 steel sheet, 0.7 mm thickness)

5. CONCLUSIONS

Due to the dispersion of the mechanical parameters, the formability of sheet metals should be described by a forming limit band. The strategy of computing such a band for the case of a DC01 steel sheet has been described in the previous sections of the paper. The theoretical model of strain localization used by the authors is an alternate formulation of Hora's criterion (MMFC).

ACKNOWLEDGMENTS

This paper was supported by the project "From micro to macro - continuum scale modeling of advanced materials in virtual fabrication ", PN II-Complex Exploratory Research Projects, financed by the Romanian Ministry of Education and Research.

REFERENCES

1. D. Banabic et al., *Sheet Metal Forming Processes: Constitutive Modelling and Numerical Simulation*, Heidelberg: Springer, 2010.
2. R. Hill, *J. Mech. Phys. Solids* **1**, 19–30 (1952).
3. H.W. Swift, *J. Mech. Phys. Solids* **1**, 1–18 (1952).
4. P. Hora, L. Tong and J. Reissner, "A prediction method for ductile sheet metal failure in FE-simulation" in *Proceedings of the Numisheet'96 Conference*, Dearborn, MI, 1996, pp. 252–256.